Getting a kick out of numerical relativity.

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ABSTRACT

Recent developments in numerical relativity have made it possible to follow reliably the coalescence of two black holes from near the innermost stable circular orbit to final ringdown. This opens up a wide variety of exciting astrophysical applications of these simulations. Chief among these is the net kick received when two unequal mass or spinning black holes merge. The magnitude of this kick has bearing on the production and growth of supermassive black holes during the epoch of structure formation, and on the retention of black holes in stellar clusters. Here we report the first accurate numerical calculation of this kick, for two nonspinning black holes in a 1.5:1 mass ratio, which is expected based on analytic considerations to give a significant fraction of the maximum possible recoil. We have performed multiple runs with different initial separations, orbital angular momenta, resolutions, extraction radii, and gauges. The full range of our kick speeds is 86–116 km s^{-1}, and the most reliable runs give kicks between 86 and 97 km s^{-1}. This is intermediate between the estimates from two recent post-Newtonian analyses and suggests that at redshifts $z \gtrsim 10$, halos with masses $\lesssim 10^9\,M_\odot$ will have difficulty retaining coalesced black holes after major mergers.

Subject headings: black hole physics — gravitational waves — relativity — cosmology: theory

1. Introduction

When two black holes merge, the gravitational waves they produce will carry away net linear momentum, barring conditions of special symmetry (e.g., two equal-mass nonspinning black holes receive no kick). The magnitude of the resulting recoil is important

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in a variety of astrophysical contexts, including the cosmological evolution of supermassive black holes (Merritt et al. 2004; Boylan-Kolchin, Ma, & Quataert 2004; Haiman 2004; Madau & Quataert 2004; Yoo & Miralda-Escudé 2004; Volonteri & Perna 2005; Libeskind et al. 2005; Micic, Abel, & Sigurdsson 2005) and the growth and retention of intermediate-mass black holes in dense stellar clusters (Taniguchi et al. 2000; Miller & Hamilton 2002a,b; Mouri & Taniguchi 2002a,b; Miller & Colbert 2004; Gültekin, Miller, & Hamilton 2004, 2006; O'Leary et al. 2006). There is therefore a long history of analytical estimates of this recoil (Peres 1962; Bekenstein 1973; Fitchett 1983; Fitchett & Detweiler 1984; Redmount & Rees 1989; Wiseman 1992; Favata, Hughes, & Holz 2004; Blanchet, Qusailah, & Will 2005; Damour & Gopakumar 2006). However, it has been shown that almost all of the recoil occurs in the strong gravity regime, inside the innermost stable circular orbit (ISCO). This is precisely where analytical treatments are least reliable. An accurate estimate of the recoil kick therefore requires full numerical simulations of the final phase of the inspiral, merger, and ringdown of the coalescence of two black holes.

Until recently, numerical simulations were not stable and accurate enough for such estimates. This situation has changed dramatically in the past year, with several groups developing codes that have allowed the evolution of binary black hole spacetimes from close to the ISCO through merger and ringdown. Most of these codes utilize techniques which allow the black holes, with their inherent singularities, to move successfully through the computational domain. One approach (Pretorius 2005, 2006) is to excise the singular region in the (physically inaccessible) interior of the black hole. In contrast, our method allows the singular region to be approximately represented in the computational domain (Baker et al. 2005, 2006; Campanelli et al. 2005b; Campanelli, Lousto, & Zlochower 2006). These new techniques have led to dramatically more effective numerical simulations of binary black hole systems, recently allowing us to determinate accurate waveforms for the final orbits and merger of equal mass systems (Baker et al. 2006), which we generalize here for nonequal masses. While some preliminary numerical calculations of recoil from mergers of non-spinning black holes have been reported (Campanelli 2005a; Herrmann, Shoemaker, & Laguna 2006), the initial separations have been too small and the resolutions too coarse for reliable and precise numbers.

Here we report the first precise fully numerical estimates of the kick received from the merger of two nonspinning black holes. We choose a 1.5:1 mass ratio because it is close to the analytically estimated optimal mass ratio for maximum kick (Fitchett 1983), but is also close enough to equal mass that resolution issues are not serious impediments to the numerical evolution. In § 2 we describe our numerical method and results, including convergence tests. We discuss the astrophysical implications of these results in § 3.

2. Numerical Simulations

Reliable simulations of binary black hole mergers require the specification of constraint-satisfying initial field data followed by stable evolution of the Einstein equations. The evolution variables hold information about gravitational fields in the form of tensor fields representing the curvature of a vacuum spacetime. Einstein's equations, together with a crucial specification of gauge (i.e., coordinate) conditions, then govern the evolution of these fields in time.

We utilize the "puncture" approach (Brandt & Brügmann 1997) to specify constraint-satisfying initial field configurations for black holes on an approximately circular inspiral trajectory within a few orbits of merger. Specifically, we consider three initial separations of coordinate distances $d_{\rm init} = 4.1M_0$, $d_{\rm init} = 6.2M_0$, and $d_{\rm init} = 7.0M_0$, where M_0 is the total initial gravitational mass of the system, and we use units where Newton's gravitational constant G and the speed of light c are set to unity so that all quantities can be represented in terms of their mass-scaling. Further, we try several slight variations in the initial angular momentum for the largest separation. These data begin our simulations as the system approaches the ISCO, allowing about 1-3 orbits before the formation of a common horizon representing the final black hole.

This data is evolved by our finite-differencing code, `Hahndol` (Imbiriba et al. 2004), on an adaptive mesh refinement structure implemented via `PARAMESH` (MacNeice et al. 2000). The punctures are allowed to move freely through the grid and are evolved according to a variant of the BSSN formulation of Einstein's equations and gauge conditions as described in Baker et al. (2006); van Meter et al. (2006).

We interpret our numerical results by studying the gravitational radiation generated by the merger which carries gauge-invariant information away from the strong-field region. In our simulations, radiation is represented by a component of the space-time Weyl curvature tensor, ψ_4. In terms of ψ_4, the time rate of change of radiated momentum can be expressed as follows (Newman & Tod 1980):

$$\frac{dP_i}{dt} = \lim_{r\to\infty}\left\{\frac{r^2}{4\pi}\int d\Omega \frac{x_i}{r}\left|\int_{-\infty}^{t} dt\psi_4\right|^2\right\} \tag{1}$$

We compute ψ_4 numerically, and extract it from the simulation data on a sphere of radius $r = 50M_0$, which we have found is sufficiently large enough to give precise results.

Our main numerical results are given in Figure 1, which shows the net speed of the center of mass as a function of time for several runs. To compare runs at different initial coordinate separations we have shifted the time axis so that the kick speed peaks at $t = 0$. Note that

these curves are representative of a much larger body of simulation data. We have varied, for example, the finest grid spacing from $h_f = M_0/32$ to $h_f = M_0/48$ for the $d_{\rm init} = 4.1M_0$ runs, and from $h_f = M_0/32$ to $h_f = M_0/40$ for the $d_{\rm init} = 7.0M_0$ runs, and in either set of runs found the kick results of different resolutions agree to within 2%, thus confirming our numerical precision. (And we have found that numerical error in these runs converges away at an acceptable rate; see (Baker et al. 2005, 2006) for previous convergence tests of our code.) We have also varied the extraction radius from $r = 30M_0$ to $r = 60M_0$ and, disallowing cases where the extraction radius significantly intersects refinement boundaries, we have found agreement across extraction radii to better than 2%. We have also varied the initial, orbital angular momentum in the $d_{\rm init} = 7.0M_0$ runs by as much as 4%, resulting in a 6% variation in the final kick values. Our full range of final kick values is 86-116 km s^{-1}. Excluding only the $d_{\rm init} = 4.1M_0$ case on the grounds that it starts with too little initial separation, our astrophysically relevant range of kick values is 86-97 km s^{-1}.

The selection of our initial momenta and centers of mass was informed by previous work on quasicircular initial data based on minimization of an effective potential (Pfeiffer 2005; Pfeiffer,Teukolsky & Cook 2000; Baker et al. 2002; Cook 1994), and further refined by trial and error. The most astrophysically relevant simulations were presumed to be those with the smoothest, monotonic radiation frequencies and time derivatives thereof. By these criteria our most reliable run gave a final kick of 92 km s^{-1}.

Also shown in Figure 1 is a post-Newtonian result for the kick, as integrated from a low-frequency cutoff. Because our simulation starts from a finite orbit, the resulting radiation has an effective low-frequency cutoff, which we have used in the post-Newtonian calculation for consistency. We integrate, specifically, a 2PN formula for the orbital frequency (Blanchet et al. 1995) and use the resulting frequency as a function of time in a 2PN formula for the radiated momentum (Blanchet, Qusailah, & Will 2005). For most of the first orbit, the agreement of the $d_{\rm init} = 7.0M_0$ simulation represented in Figure 1 with the post-Newtonian calculation is better than 1%.

Recently, two groups have made refined analytic estimates of the kick from the merger of nonspinning black holes, both with precise answers but differing in magnitude by a factor of three. Blanchet, Qusailah, & Will (2005) predict a speed of 155$\pm$25 km s^{-1} for a 1.5:1 mass ratio, whereas Damour & Gopakumar (2006) predict 48 km s^{-1}. Our result is between these estimates but inconsistent with either one. We note, however, that Blanchet, Qusailah, & Will (2005) integrate only until the horizons overlap, which is close to the peak of our kick. At that point, our most reliable run (i.e., with the widest initial separation and least apparent eccentricity) gives a kick of $\approx$ 150 km s^{-1}, which is encouragingly close to the analytic estimate. This points out the importance of the $\sim$ 40% post-peak reduction in the

kick. This reduction can also be seen in Figure 5 of Damour & Gopakumar (2006). We attribute it to the evolution by more than π radians of the phase of the emitted momentum during the merger, which thus partially opposes the vector kick that exists at the time of horizon overlap. It is also noteworthy that, integrating only up to ISCO, Blanchet, Qusailah, & Will (2005) obtain a kick of ≈ 14 km s^{-1} for a mass ratio of 1.5:1, and in our most reliable run we also obtain a kick of ≈ 14 km s^{-1} at ISCO.

An initial numerical estimate for 1.04:1 and 1.18:1 mass ratios was made by Herrmann, Shoemaker, & L (2006) (albeit at much lower resolution and starting much closer than our simulations), and they find kick speeds of 9 km s^{-1} and 33 km s^{-1}, respectively. One can compare results at different mass with reference to the Fitchett (1983) fitting formula kick $\propto q^2(q-1)/(1+q)^5 = (\mu/m)^2 \mathrm{d}m/m$, where $q = m_1/m_2 > 1$ is the mass ratio, $\mu/m = m_1 m_2/(m_1+m_2)^2$ is the symmetric mass ratio and $\mathrm{d}m/m = (m_1 - m_2)/(m_1 + m_2)$ is the fractional mass difference. Although originally derived in the context of a leading order post-Newtonian approximation, this formula is suggested to be a good approximation by perturbative Schwarzschild calculations (Fitchett & Detweiler 1984), and it also closely agrees with more recent post-Newtonian results (Blanchet, Qusailah, & Will 2005; Damour & Gopakumar 2006). By Fitchett's formula, the Herrmann, Shoemaker, & Laguna (2006) results would imply ~ 80 km s^{-1} for a mass ratio of 1.5:1, closer to our final value than the analytic estimates.

3. Discussion and Conclusions

Our range of 101 ± 15 km s^{-1} (and best estimate of 92 ± 6 km s^{-1}) for the kick received in a 1.5:1 merger of nonspinning black holes has important implications for the assembly of supermassive black holes in the early universe (see also Merritt et al. 2004). This is because as dark matter halos merge in the process of hierarchical structure assembly, their central black holes are also presumed to merge if the halo mass ratio is not too extreme (otherwise dynamical friction on the halos is likely to be inefficient; see Taffoni et al. 2003). If black hole masses are linked to the mass of their host halos, it is therefore expected that in the early universe, black hole mergers are likely to involve comparable-mass objects. If the resulting kick exceeds the escape speed of the merged dark matter halo, the halo is left without a black hole. This could, therefore, have a significant impact on the number of black hole mergers in the early universe. In addition, if mergers between comparable-mass halos are common at redshifts $z \gtrsim 10$, these ejections might reduce substantially the fraction of halos that host black holes.

A full appraisal of the consequences of recoil will require detailed numerical studies of the effects of black hole spin and mass ratio, which will be our subject in future papers.

We can make initial guesses by focusing on nonspinning black holes and by adopting as before the Fitchett (1983) formula for the dependence of kick speed on mass ratio. Setting $v_{\rm kick}(q=1.5)=92$ km s^{-1} fixes the curve.

To estimate the escape speed from a dark matter halo, we follow the treatment of Merritt et al. (2004). The escape speed from a halo of virial mass $M_{\rm vir}$ is $V_{\rm esc}^2=2cg(c)M_{\rm vir}/R_{\rm vir}$, where as before we set $G=1$. Here c is the concentration parameter for the halo, $g(c)=[\ln(1+c)-c/(1+c)]^{-1}$, and we define $R_{\rm vir}$ as the radius inside which the average density is $\Delta_c=18\pi^2+82x-39x^2$ for a flat universe, where $x\equiv\Omega_M(z)-1$ (see Bryan & Norman 1998, equation 6). The critical density for a flat universe is $\rho_{\rm crit}=3H_0^2/(8\pi)[\Omega_M(1+z)^3+\Omega_\Lambda]\approx 10^{-29}$ g cm$^{-3}(1+z)^3$ (e.g., Peacock 1999, section 3.2). We use $\Omega_M=0.27$ and $\Omega_\Lambda=0.73$. ¿From equation (18) of Bullock et al. (2001), the concentration parameter scales as $c=9\,[M_{\rm vir}/(2\times10^{13}\,M_\odot)]^{-0.13}\,(1+z)^{-1}$.

In Figure 2 we plot the minimum halo mass as a function of redshift such that $V_{\rm esc}>v_{\rm kick}$ for the listed mass ratios of 1.5:1, 3:1, 5:1, and 10:1, as projected from our results by the Fitchett scaling. Note that the low power of $M_{\rm vir}$ in the escape speed means that the minimum halo mass depends sensitively on the kick speed. For example, if the kick speed at a 1.5:1 mass ratio were 200 km s^{-1} instead of 92 km s^{-1}, the threshold halo mass at $z=10$ for this mass ratio would jump from $4\times10^8\,M_\odot$ to $3\times10^9\,M_\odot$. This would in turn reduce the number density of halos massive enough to retain their black holes by a factor ~ 20 (see Mo & White 2002 for a pedagogical discussion of how to estimate halo number densities). This points out the importance of estimating kick speeds accurately.

As discussed by Merritt et al. (2004), kicks are also important in the current-day universe for low-mass concentrations of stars. Their Figure 2 is a useful summary of escape speeds from the centers of galaxies and globular clusters. From their figure, we see that comparable-mass mergers with kick speeds ~ 100 km s^{-1} will cause ejection from globulars and some dwarf galaxies, but that more massive galaxies will retain the remnant. Using the Fitchett (1983) scaling with mass ratio, we find that black holes that are $\gtrsim 10$ times more massive than their companions are not ejected from globular clusters with escape speeds of a few tens of km s^{-1}. Coincidentally, this is approximately the same mass ratio required to protect a massive black hole from cluster ejection from three-body interactions (Gültekin, Miller, & Hamilton 2004, 2006). Therefore, if an intermediate-mass black hole of $>100-200\,M_\odot$ is formed in a cluster, it can stay and potentially grow through future interactions. Stellar-mass binary black holes ($M<50\,M_\odot$) will be ejected from such clusters prior to merger by Newtonian three-body interactions (Kulkarni, Hut, & McMillan 1993; Sigurdsson & Hernquist 1993), hence recoil is not so important for low-mass black holes in this context. However, mergers of black hole binaries in low-density galactic disks could

produce a population of high-speed coalesced black holes.

In conclusion, we have presented a reliable and precise, fully numerical estimate of the gravitational recoil produced by the merger of two unequal mass nonspinning black holes. Our best estimate of 92±6 km s^{-1} for a 1.5:1 mass ratio is intermediate between the recent analytic estimates of Blanchet, Qusailah, & Will (2005), who suggest 155±25 km s^{-1} for this mass ratio, and Damour & Gopakumar (2006), whose formulae would imply 48 km s^{-1}. Our results are thus an important step in accurately evaluating the astrophysical consequences of gravitational radiation recoil in dense stellar clusters and the early universe.

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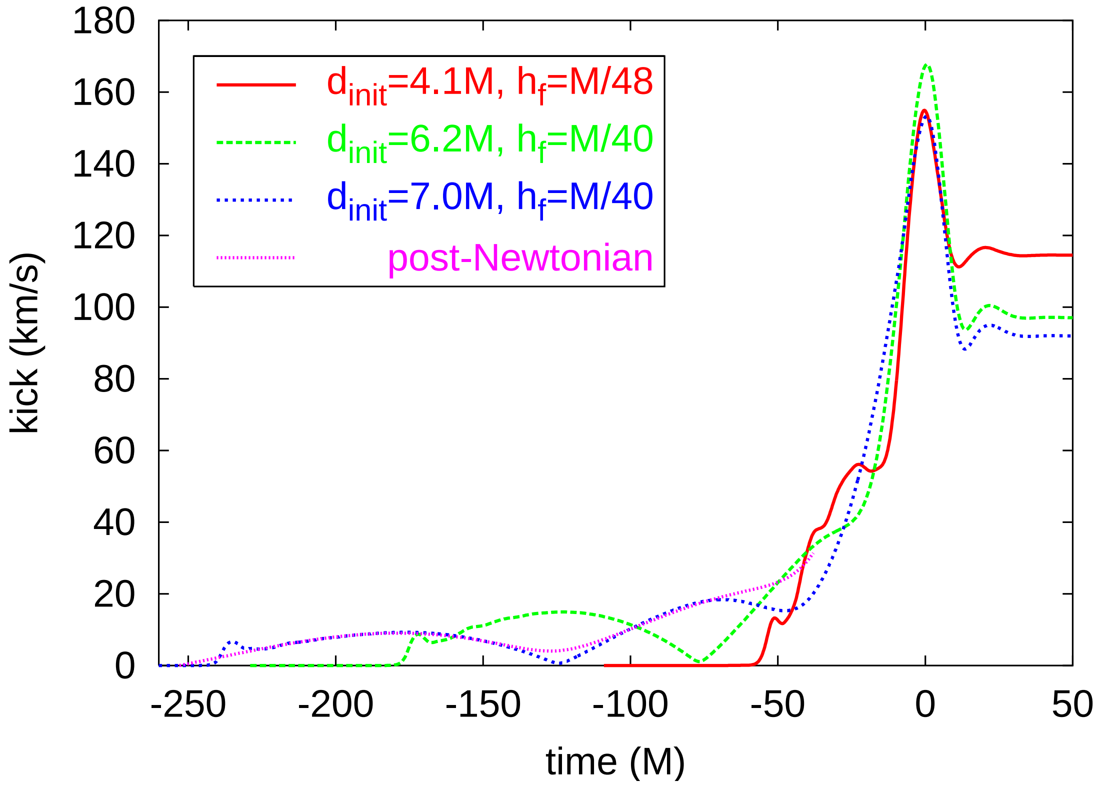

Fig. 1.— The magnitude of the radiated momentum, as a function of time, from three different simulations. For initial coordinate separations of $d_{\rm init} = 4.1M_0, 6.2M_0,$ and $7.0M_0$, the final values of the kicks are respectively 113 km/s, 97 km/s, and 92 km/s. Also shown is the 2nd order post-Newtonian radiated momentum, which was computed from a low frequency cutoff commensurate with that of the $d_{\rm init} = 7.0M_0$ simulation (see text for details). The excellent agreement of the post-Newtonian kick with that of the $d_{\rm init} = 7.0M_0$ simulation over most of the first orbit, together with the agreement to within 6% of the final kick from the $d_{\rm init} = 7.0M_0$ simulation with that of the $d_{\rm init} = 6.2M_0$ simulation, lends support to the accuracy of these results.

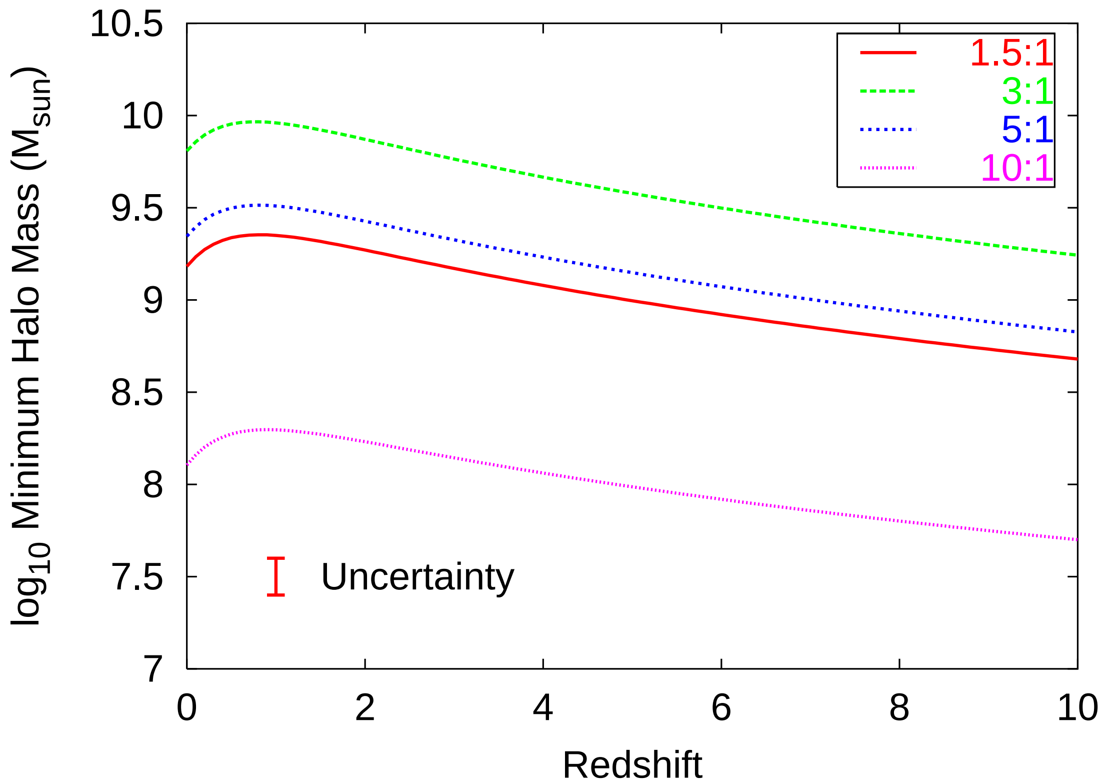

Fig. 2.— Minimum mass of a dark matter halo at a given redshift required to retain the product of the merger of two nonspinning black holes with a mass ratio indicated on the curve. Details of the computation are in the text. Note that we use the Fitchett (1983) analytical estimate of the mass ratio dependence, assuming our numerical result of 92 km s^{-1} for the mass ratio of 1.5:1. Then a mass ratio of 3:1 gives 140 km s^{-1}, a mass ratio of 5:1 gives 103 km s^{-1}, and a mass ratio of 10:1 gives 45 km s^{-1}. This figure indicates that early halos might lose merger remnants because of the kick, but in the current universe only globular clusters or the smallest dwarf galaxies could have black holes ejected because of gravitational radiation recoil.